Final Exam

DATA ANALYSIS A - FOY28 A a.a. 2025-26

March 2, 2026

Ex. 1 (Data and descriptive statistics, 10-13 min)

The following numbers are the playing times (in seconds) of a random selection of pop songs:

$$448, 242, 231, 246, 246, 293, 280, 227, 244, 213, 262, 239, 213, 258, 255, 257$$

a) Calculate the mean and median.

b) Calculate the variance and standard deviation.

c) Calculate the uncertainty on the mean.

d) Plot the distribution of playing times using an appropriate bin size.

SOLUTION

a) $\mu = 259.625$; $median = 246$; b) $\sigma^2 = 2789.6$; $\sigma = 52.8$; c) $\sigma_\mu = 13.2$.

Ex. 2 (Weighted mean, 7 min)

You measure 3 times the temperature of liquid nitrogen in degrees Celsius $°C$. Calculate the weighted mean.

$$T_1 = -196.1 \pm 0.1; T_2 = -194.2 \pm 0.4; T_3 = -195.5 \pm 0.7$$

SOLUTION

$\mu_w = (-195.979 \pm 0.096)°C = (-196.0 \pm 0.1)°C$

Ex. 3 (Significant figures, 5 min)

Report the following results correctly, by stating the uncertainty to only 1 significant digit.

a) 724.41 ± 0.37

b) 0.0069 ± 0.0019

c) 2041 ± 0.51

d) 65.1 ± 20

e) 0.10101 ± 0.01010

f) 6.4192 ± 0.099

SOLUTION

a) 724.4 ± 0.4; b) 0.007 ± 0.002; c) 2041.0 ± 0.5; d) $70 \pm 20 = (7 \pm 2) \cdot 10^1$; e) 0.10 ± 0.01; f) 6.4 ± 0.1

Ex. 4 (Covariance and Correlation, 10 min)

Consider the following data. Calculate the covariance and the Pearson correlation coefficient between variables x and y. What can you say about x and y ?

SOLUTION

$\mu_{bread} = 300$; $\mu_{flour} = 4250$; $\sigma_X = 158.11$; $\sigma_Y = 2772.63$; $cov(X, Y) = -325000.0$; $\rho = -0.7413$.

Cost of Bread ($)	100	200	400	500
Flour Availability (ton)	9000	3000	2000	3000

Ex. 5 (Poisson, 5 min)

A small restaurant receives an average of 15 customers per day for lunch. Find the probability that the restaurant receives:

a) Exactly 8 customers.

b) Less than 2 customers.

SOLUTION
a) $p(k=8,\lambda) = \frac{\lambda^k \cdot e^{-\lambda}}{k!} = \frac{15^8 \cdot e^{-15}}{8!} = 0.0194$. b) $P(0)+P(1) = \frac{15^0 \cdot e^{-15}}{0!} + \frac{15^1 \cdot e^{-15}}{1!} = 4.89 \cdot 10^{-6}$.

Ex. 6 (Binomial, 7 min)

Is it unusual to see less than 3 heads in 12 flips of a coin? Why?
SOLUTION
p = 0.5, n=12. $P(k<3,p,n) = P(0)+P(1)+P(2)$.
$P(0) = p^k(1-p)^{n-k}\frac{n!}{k!(n-k)!} = 0.5^{12} = 0.000244141$.
$P(1) = p^k(1-p)^{n-k}\frac{n!}{k!(n-k)!} = 0.5 \cdot (0.5)^{11} \cdot 12 = 0.002929688$.
$P(2) = p^k(1-p)^{n-k}\frac{n!}{k!(n-k)!} = 0.5^2 \cdot (0.5)^{10} \cdot \frac{12 \cdot 11}{2} = 0.016113281$.
$P(0)+P(1)+P(2) = 0.01928711$. Yes, it is unusual because the probability is very small.

Ex. 7 (Bayes, 7 min)

There are two boxes containing black and green stones. First box has 2 black and 2 green stones and the second box has 4 black and 1 green stones. A stone is picked randomly from one of the boxes and is found to be green. What is the probability that the stone is drawn from the second box?
SOLUTION
$P(2|green) = \frac{P(green|2)P(2)}{P(green)} = \frac{P(green|2)P(2)}{P(green|1)P(1)+P(green|2)P(2)} = \frac{1/5 \cdot 1/2}{2/4 \cdot 1/2 + 1/5 \cdot 1/2} = \frac{1/10}{1/4+1/10} = \frac{1/10}{7/20} = 2/7$

Ex. 8 (Gaussian, 7 min)

Suppose temperature readings are distributed according to a Gaussian distribution with a mean $\mu = 56$ C and standard deviation $\sigma = 4$ C. Find the probabilities that a random temperature reading x:

a) Is exactly 56, i.e. $p(x=56)$?

b) Is greater than 64.2, i.e. $p(x > 64.2)$?

c) Is between 52.0 and 60.0, i.e. $p(52 < x < 60)$?

SOLUTION
a) The probability is zero. In fact, $x = 56$ corresponds to $z = 0$, and from the integral table we can read that the integral is zero. The probability is the integral below the curve. It would be not correct to evaluate the function at x=56, i.e. $p(x=56) = \frac{1}{\sigma\sqrt{2\pi}}$.
b) $z = \frac{x-\mu}{\sigma} = \frac{64.2-56}{4} = 2.05$; $p(z > 2.05) = (1 - p(-2.05 < z < 2.05))/2 = (1-0.9596)/2 = 0.0202$.
c) $z_1 = -1$, $z_2 = 1$, $p(52 < x < 60) = p(-1 < z < 1) = 0.6827$.

Ex. 9 (Uniform, 6 min)

The smiling times, in seconds, of a eight-week-old babies are uniformly distributed between 0 and 23 seconds.

a) Draw the distribution of smiling times.

b) What is the mean and standard deviation of the distribution ?

c) What is the probability that an eight-week-old baby smiles between 2 and 18 seconds?

SOLUTION

a) $p = const = 1/(b-a) = 1/23$.
b) $\mu = (a+b)/2 = 23/2 = 11.5$s. $\sigma = (b-a)/\sqrt{12} = 23/\sqrt{12} = 6.64$s.
c) $p(2 < x < 18) = (18-2) \cdot 1/23 \approx 0.695 = 69.5\%$

Ex. 10 (Error Propagation, 5 min)

On the moon, the weight P (expressed in Newton) is given by $P = m \cdot g$, where m is the mass (expressed in kg) and g is the (constant) gravity acceleration, $g = 1.62 m/s^2$. Calculate the weight of an astronaut of mass $m = (80 \pm 1) kg$.

SOLUTION

$P = (m \cdot g \pm \sigma_m \cdot g) N = (129.6 \pm 1.62) N \approx (130 \pm 2) N$.